

## Fitting oscillating string gas cosmology to supernova data

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## Fitting oscillating string gas cosmology to supernova data

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**ABSTRACT:** In string gas cosmology, extra dimensions are stabilised by a gas of strings. In the matter-dominated era, competition between matter pushing the extra dimensions to expand and the string gas pulling them back can lead to oscillations of the extra dimensions and acceleration in the visible dimensions. We fit this model to supernova data, taking into account the Big Bang Nucleosynthesis constraint on the energy density of the string gas. The fit to the Union set of supernova data is acceptable, but the fit to the ESSENCE data is poor.

**KEYWORDS:** Strings and branes phenomenology

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## 1 Introduction

**String gas cosmology.** String gas cosmology is a cosmological scenario motivated by string theory [1–3] (see [4, 5] for reviews and [6] for another perspective). In the original formulation of string gas cosmology, all spatial dimensions are treated on an equal footing: they are all toroidal and start out at the string size. The aim is that dynamical processes in the early universe will allow only three dimensions to expand to macroscopic size, while the extra dimensions are stabilised at the string size by a gas of strings. Assuming that the dilaton is stabilised by some other mechanism, the string gas can stabilise the extra dimensions during the radiation-dominated era [7, 8] (see also [1, 9–16]). However, when the universe becomes matter-dominated, the matter will push the extra dimensions to open up [1, 7, 8, 17]. It was shown in [18] that the gas of strings can still prevent the extra dimensions from growing too large, but they cannot be completely stabilised. There is a competition between the push of matter and the pull of strings. If the number density of the strings is too small, the extra dimensions will grow to macroscopic size. If the strings win, the size of the extra dimensions will undergo damped oscillations around the self-dual radius. The oscillations between expansion and contraction of the extra dimensions induce oscillations in the expansion rate of the large dimensions, which can involve alternating periods of acceleration and deceleration [18]. (This kind of a mechanism has also been studied in [19].)

Since the oscillations can start only after the universe becomes matter-dominated, they provide an in-built mechanism for late-time acceleration in string gas cosmology, one that alleviates the coincidence problem in a manner similar to scaling and tracker fields [20, 21]. The mechanism is based on ingredients already present in string gas cosmology and does not require adding new degrees of freedom or turning on new interactions. However, the oscillating expansion history is quite different from the  $\Lambda$ CDM model which is known to be a good fit to the observations. (For comparison of some oscillating models to observations, see [22, 23].)

We compare the model studied in [18] to observations of type Ia supernovae (SNe Ia), using the Big Bang Nucleosynthesis (BBN) constraint on new radiation degrees of

freedom. In section 2 we describe the string gas model, and in section 3 we fit the model to the Union and ESSENCE sets of SNIa data. The distances predicted by the model provide an acceptable fit to the Union data, but the fit to the ESSENCE data is poor. In section 4 we summarise our results and discuss how to make the model more realistic.

## 2 The string gas model

**The metric and the equation of motion.** We consider the string gas model discussed in [18]. The spacetime is ten-dimensional, with the metric

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1}^3 dx^i dx^i + b(t)^2 \sum_{j=1}^6 dx^j dx^j, \quad (2.1)$$

where  $i = 1 \dots 3$  labels the visible dimensions and  $j = 1 \dots 6$  labels the extra dimensions. All spatial dimensions are taken to be toroidal. We take the value  $b = 1$  to correspond to extra dimensions at the self-dual radius given by the string length  $l_s \equiv \sqrt{\alpha'}$ .

We assume that the dilaton has been stabilised [10–14] in a way that leaves the equation of motion of the metric unconstrained, so that it reduces to the Einstein equation

$$G_{\mu\nu} = \kappa^2 T_{\mu\nu}, \quad (2.2)$$

where  $G_{\mu\nu}$  is the Einstein tensor,  $\kappa^2$  is the 10-dimensional gravitational coupling and  $T_{\mu\nu}$  is the energy-momentum tensor. (We take the cosmological constant to be zero.)

Given the symmetries of the metric (2.1), the energy-momentum tensor has the form

$$T_{\nu}^{\mu} = \text{diag}(-\rho(t), p(t), p(t), p(t), P(t), P(t), P(t), P(t), P(t), P(t)), \quad (2.3)$$

where  $p$  and  $P$  are the pressure in the visible dimensions and the extra dimensions, respectively. With (2.1) and (2.3), the Einstein equation (2.2) reads

$$\kappa^2 \rho = 3H_a^2 + 18H_a H_b + 15H_b^2 \quad (2.4)$$

$$\dot{H}_a + H_a^2 = -\frac{1}{6}\kappa^2(\rho + 3p) - \frac{3}{8}\kappa^2(\rho - 3p + 2P) + 6H_a H_b + 10H_b^2 \quad (2.5)$$

$$\kappa^2(\rho - 3p + 2P) = 8\dot{H}_b + 24H_a H_b + 48H_b^2, \quad (2.6)$$

where  $H_a \equiv \dot{a}/a$  is the expansion rate of the visible dimensions and  $H_b \equiv \dot{b}/b$  is the expansion rate of the extra dimensions.

**The distance.** In Friedmann-Robertson-Walker models, the luminosity distance  $D_L$  is determined by the Hubble rate as a function of redshift and the spatial curvature at one time,

$$D_L = (1+z) \frac{1}{\Omega_{K0} H_{a0}} \sinh \left( \Omega_{K0} H_{a0} \int_0^z \frac{dz'}{H_a(z')} \right), \quad (2.7)$$

where  $\Omega_K$  is the spatial curvature density parameter and the subscript 0 refers to the present day (see e.g. [24, 25]).

The metric (2.1) is not homogeneous and isotropic, so the relation (2.7) does not hold. The distance should instead be calculated from the general equation (the null geodesic shear has been neglected)

$$\partial_\lambda^2 D_A = -\frac{1}{2} G_{\mu\nu} k^\mu k^\nu D_A, \quad (2.8)$$

where  $D_A = (1+z)^{-2} D_L$  is the angular diameter distance,  $\partial_\lambda$  is the derivative along the null geodesic and  $k^\mu$  is the photon momentum (see e.g. [25]). We only consider light rays which propagate in the visible directions, and (2.8) reduces to

$$H_a \partial_z \left[ (1+z)^2 H_a \partial_z D_A \right] = \left( \dot{H}_a + 3\dot{H}_b - 3H_a H_b + 3H_b^2 \right) D_A. \quad (2.9)$$

Only if  $H_b = 0$  can we integrate (2.9) to recover (the spatially flat case of) (2.7). The relation (2.7) was formulated as a consistency check for the FRW metric in [24]. String gas cosmology provides a concrete example of a model where the metric does not have the FRW form and the consistency condition is violated. In general, this is also the case in other models with dynamical extra dimensions. However, in models where the observers are confined to a brane, distances along the visible directions are calculated with the induced metric on the brane, and the evolution of the extra dimensions does not directly enter the light propagation equation (2.8).

**The matter content.** In addition to ordinary four-dimensional radiation ( $\gamma$ ) and pressureless matter ( $m$ ), we have a gas of massless strings ( $s$ ) with winding and momentum modes in the extra dimensions and momentum modes in the visible dimensions. The contribution of radiation and matter to the energy-momentum tensor (2.3) is

$$\rho_\gamma = \rho_{\gamma,in} a^{-4} b^{-6}, \quad p_\gamma = \frac{1}{3} \rho_\gamma, \quad P_\gamma = 0 \quad (2.10)$$

$$\rho_m = \rho_{m,in} a^{-3} b^{-6}, \quad p_m = 0, \quad P_m = 0, \quad (2.11)$$

and for the string gas we have [18]

$$\rho_s = M^{-1} \rho_{s,in} a^{-3} b^{-6} \sqrt{M^2 a^{-2} + b^{-2} + b^2 - 2} \quad (2.12)$$

$$p_s = \frac{1}{3} \frac{M^2 a^{-2}}{M^2 a^{-2} + b^{-2} + b^2 - 2} \rho_s \quad (2.13)$$

$$P_s = \frac{1}{6} \frac{b^{-2} - b^2}{M^2 a^{-2} + b^{-2} + b^2 - 2} \rho_s, \quad (2.14)$$

where the subscript *in* refers to the initial values, and  $M$  is the initial momentum of a string in the visible directions in units of the string scale  $l_s^{-1}$ . Note that all strings are assumed to have the same momentum.

There are four parameters in the energy-momentum tensor: the scale  $M$  and the energy densities  $\rho_{\gamma,in}$ ,  $\rho_{m,in}$  and  $\rho_{s,in}$ . However, the parameter  $M$  only determines the absolute scale, and does not affect the dynamics, as we see by rescaling  $a \rightarrow Ma$ . The evolution of

the system therefore depends on only two dimensionless combinations of the parameters, which we take to be the following:

$$\begin{aligned} r &\equiv M^{-1} \frac{\rho_{\gamma,in}}{\rho_{m,in}} \\ f_s &\equiv \frac{\rho_{s,in}}{\rho_{\gamma,in}} . \end{aligned} \tag{2.15}$$

Now, with the rescaled  $a$ , the total energy density reads

$$\rho = \rho_{m,in} M^{-3} a^{-3} b^{-6} \left( 1 + r a^{-1} + r f_s \sqrt{a^{-2} + b^{-2} + b^2 - 2} \right) , \tag{2.16}$$

and the pressures are written accordingly.

Deep in the radiation-dominated era (in particular, during BBN), the energy density of the string gas evolves like radiation, and contributes to the total energy density a fraction  $\Omega_{s,in} = f_s/(1 + f_s)$ , given that the contribution of matter is negligible and  $b = 1$  in the radiation-dominated era. The string fraction  $f_s$  is related to the effective number of additional neutrino species  $\Delta N_\nu$  by  $f_s = 7\Delta N_\nu/43$  [26]. From BBN we have, assuming negligible neutrino chemical potential, the constraint  $\Delta N_\nu \leq 1.5$ , giving  $f_s \leq 0.24$ , or  $\Omega_{s,in} \leq 0.20$  [27]. Allowing for a large electron neutrino chemical potential, we have  $\Delta N_\nu \leq 4.1$ , which translates into  $f_s \leq 0.7$ , or  $\Omega_{s,in} \leq 0.4$  [28]. The bound depends on the assumption that the gravitational coupling during BBN is the same as today, which is not necessarily true in the string gas model, since  $G_N \propto b^{-6}$ . If  $b < 1$  today, the gravitational coupling at BBN is reduced relative to the present value, so there is more room for new degrees of freedom. However, generally  $b$  dips below unity only very slightly, and typically  $b > 1$  today, so taking this into account would make the constraints tighter. It was observed in [18] that a requirement for the string gas being able to keep the extra dimensions small is  $r f_s > 3/2$ . There are no other constraints on  $r$ , since it depends on  $M$ , the initial momentum of the strings in the visible directions, on which there is no limit.

The string gas behaves like a scaling solution [20] in the radiation-dominated era and like a tracker solution [21] in the matter-dominated era [18]. The value  $b = 1$  is an attractor point: as long as the initial value of  $b$  is not too large ( $b < \sqrt{2}$  is a necessary condition),  $b$  will rapidly evolve to unity, and the extra dimensions are stable. Then the energy density of the string gas behaves exactly like radiation. When the universe becomes matter-dominated, the string gas starts tracking the matter as the extra dimensions expand. When the extra dimensions are pulled back and contracted by the strings, the visible dimensions start oscillating between deceleration and acceleration. (If the string gas is too weak to prevent the extra dimensions from opening up, they will grow without bound, and there will be no acceleration in the visible dimensions. We are not interested in this possibility.)

### 3 Comparison with observations

**The observations.** We want to see how well the expansion history of the system of equations (2.4)–(2.6) with the energy density (2.16), and the distance given by (2.9), agrees with cosmological observations. The interpretation of many observations such as the cosmic

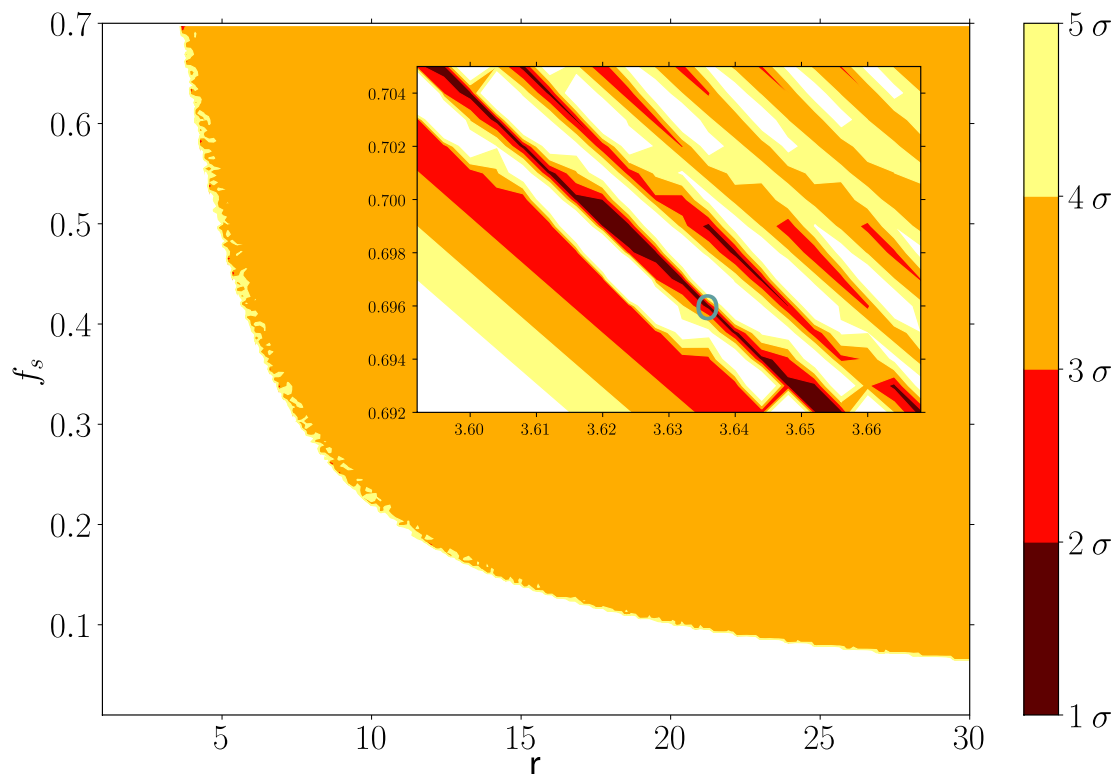
microwave background (CMB) and the baryon acoustic oscillations requires perturbation theory. The effect of the string gas and the extra dimensions on the perturbation equations has not been completely worked out [29–31], so we will consider only observations which are independent of perturbations. Two important sets of observations which depend only on the background are luminosity distances of SNe Ia and the primordial abundance of light elements. The ages of passively evolving galaxies also provide a measurement of the Hubble parameter as a function of redshift independent of the distance scale [32]. In addition, there are local measurements of the Hubble parameter, the age of the universe and the matter density.

We will use the ESSENCE SNIa dataset [33] and the Union compilation [34] separately. The Union dataset is the newest and most comprehensive collection, but it has been analysed with the assumption that the  $\Lambda$ CDM model is correct. Therefore the results cannot, strictly speaking, be used to compare between cosmological models in an unbiased manner, especially in the case of models which are significantly different from  $\Lambda$ CDM, like the string gas model. Therefore, we also fit to the ESSENCE dataset, which has been analysed differently, for comparison. We find that, despite the bias in the Union analysis, the string gas model fits the Union dataset better than the ESSENCE data. This could be due to more conservative treatment of errors in the Union analysis.

We will also take into account the BBN constraint on new radiation degrees of freedom from the observed abundance of light elements [28]. We will not use the data on the ages of passively evolving galaxies, due to possible systematic effects related to stellar evolution. In addition to above data, a number of other general dark energy probes have also been suggested. In particular, the baryon acoustic oscillations [35] and the CMB shift parameter [36] have been considered as standard rulers. However, both of these probes suffer from model dependence and caution should be exercised when applying them to models other than  $\Lambda$ CDM [37, 38].

**The supernova datafit.** We use a grid method to scan the model parameter space  $(r, f_s)$ , because the complicated confidence contour structure makes Monte Carlo Markov Chain methods ineffective. We refined the grid until, for a typical size of  $400 \times 400$ , the fit no longer improved significantly. In order to determine the best fit values, we further zoomed into regions with high values of  $\chi^2$ . When we do not apply the BBN constraint  $f_s < 0.7$ , we restrict the scan to  $f_s < 9$ , corresponding to  $\Omega_{s,in} < 0.9$ .

In figure 1 we plot the goodness-of-fit in the  $(r, f_s)$ -plane for the Union dataset (the behaviour is similar for the ESSENCE data). The inset shows the region around the best-fit model, marked with a circle. The  $\chi^2$  contours have a striking structure. The lines of equal  $\chi^2$  are disjoint, and nearby points can have radically different values of goodness-of-fit. This is not an artifact of the analysis. (A complicated confidence level contour structure for an oscillating model was also found in [23].) In order to have enough acceleration in the visible dimensions at sufficiently late times, the present day has to be in a specific location, just after the rise of one of the first few oscillations. The details of the oscillations, in turn, depend on  $r$  and  $f_s$  in a complex manner. (Note that the late-time evolution depends on the parameter  $r$  only via the initial conditions, as the radiation term in the energy



**Figure 1.** Confidence level contours in the  $(r, f_s)$ -plane for the Union dataset. The best-fit model is marked with a circle.

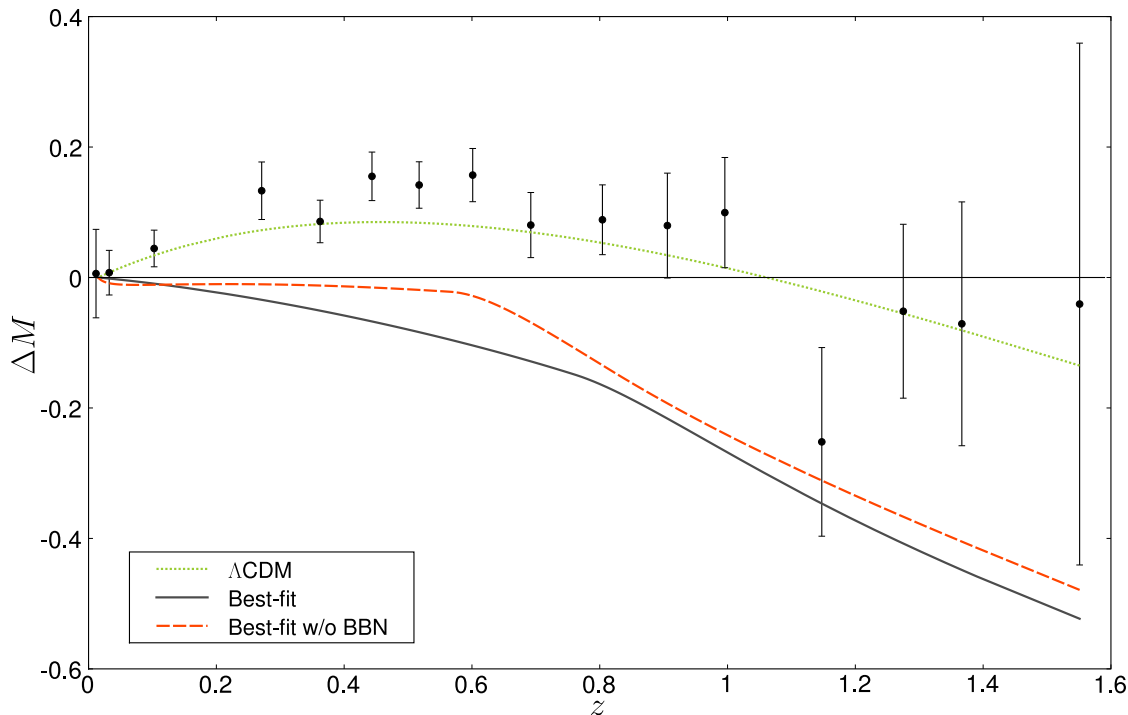
Model	Dataset	$\chi^2_{bf}$	$p(\%)$	$r$	$f_s$	Remarks
$\Lambda$ CDM	Union	308.3	44	-	-	$\Omega_\Lambda = 0.68$
$\Lambda$ CDM	Essence	196.0	37	-	-	$\Omega_\Lambda = 0.75$
String gas	Union	329.8	15	3.636	0.696	$f_s \leq 0.7$
String gas	Union	317.6	28	0.893	3.246	$f_s \leq 9$
String gas	Essence	262.2	0.03	3.636	0.696	$f_s \leq 0.7$
String gas	Essence	234.5	1	0.833	3.497	$f_s \leq 9$

**Table 1.** Goodness-of-fit and best-fit parameters for  $\Lambda$ CDM and the string gas model for different datasets, with and without the BBN constraint.

density (2.16) is negligible at late times.) Also, in order to have strong acceleration, the extra dimensions have to expand almost to the point of not turning back, and then contract rapidly. If the extra dimensions were to expand slightly more, they would not turn around, and there would be no acceleration. Therefore, the best fits are obtained on the border of very poor fits, as seen in figure 1.

In table 1 we give the parameter values as well as the  $\chi^2$  (and the corresponding probability  $p$  of obtaining the data given the model) for the best-fit string gas model with and without the BBN constraint for both datasets; values for the best-fit  $\Lambda$ CDM model



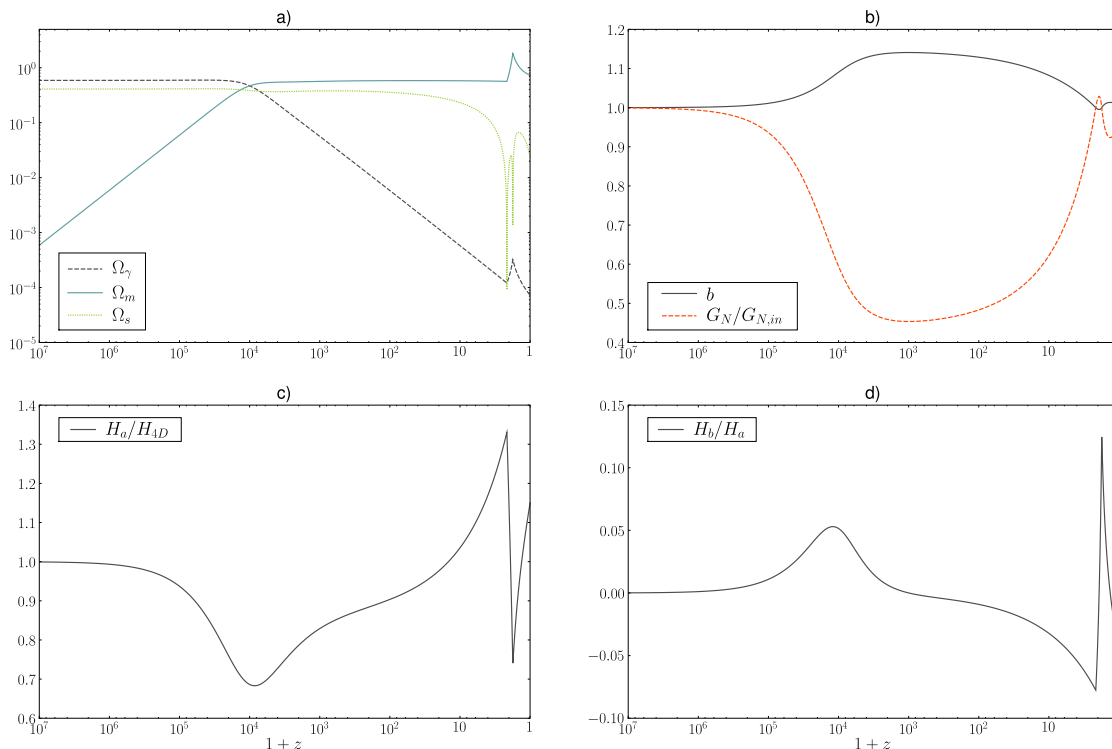


**Figure 2.** Magnitude as a function of redshift compared to an empty universe for the Union data, for the string gas model with the BBN constraint and the  $\Lambda$ CDM model.

are shown for comparison. Because of the complex dependence of the goodness-of-fit on  $r$  and  $f_s$ , we cannot definitely rule out the possibility that there would be a better fitting model somewhere in the regions that we did not completely study. In the patches that we did cover in detail, the quality of fit is already saturated at the scale visible in figure 1 and does not improve when zooming to smaller regions.

For the Union dataset, the  $\chi^2$  for the best-fit string gas model without the BBN constraint is 9.3 points worse than for the  $\Lambda$ CDM model, and 21.5 points worse when the BBN constraint is taken into account. For the ESSENCE data, we have  $\Delta\chi^2 = 40.2$  and 66.2, respectively. In figure 2 we show the distance-redshift relation for the best-fit model to the Union dataset with and without the BBN constraint. The string gas behaviour is clearly different from the  $\Lambda$ CDM model, and provides a worse fit to the data, though for the Union data, the quality of the fit is still good. The string gas model would be further disfavoured if we took into account that it has one extra parameter compared to  $\Lambda$ CDM.

In figure 3, we plot some quantities for the best-fit model to the Union dataset (with the BBN constraint included). In figure 3 a), we show the density parameters of radiation, matter and the string gas. The energy density of the string gas is completely subdominant at late times,  $\Omega_{s0} = 0.02$ . However, the string gas can still have a large impact on the dynamics, because its energy-momentum tensor (2.12)–(2.14) violates the null energy condition. When the expansion is faster than in the Einstein-de Sitter case, the matter density parameter  $\Omega_m \equiv \kappa^2 \rho_m / (3H_a^2)$  is smaller than unity, and in principle it could be in



**Figure 3.** a) Density parameters  $\Omega_i \equiv \kappa^2 \rho_i / (3H_a^2)$ , b) size of the extra dimensions and Newton’s constant, c) expansion rate of the large dimensions ( $H_{4D}$  is the Hubble parameter in the usual four-dimensional case) and d) expansion rate of the extra dimensions, for the best-fit model to the Union data, with the BBN constraint.

the observationally allowed range  $\Omega_{m0} \approx 0.2-0.3$  today. However, for the best-fit model we have  $\Omega_{m0} = 0.73$ , far too large.

In figure 3 b) we show the scale factor of the extra dimensions  $b$  and the four-dimensional gravitational coupling  $G_N \propto b^{-6}$ . The difference between  $b$  at BBN and today is small, and well within the observational limits discussed in [18]. However,  $b$  deviates noticeably from unity at last scattering at  $z = 1100$ :  $b_{LS} = 1.14$ ,  $G_{N,LS}/G_{N,in} = 0.45$ . This is a generic feature of the string gas model, because last scattering is soon after the matter-radiation equality, when the extra dimensions start opening up. This prediction could provide a stringent constraint. However, quoted limits on the variation of  $G_N$  (or on new radiation degrees of freedom) from the CMB and other non-BBN probes are model-dependent [38, 39], and rely on perturbation theory. (Note that the string gas does not behave like radiation at last scattering.)

In figure 3 c) we show the expansion rate of the visible dimensions  $H_a$  relative to what it would be without the extra dimensions and the string gas, denoted by  $H_{4D}$ . (In the matter-dominated era,  $H_{4D} = 2/(3t)$ .) Comparing to the plot of  $H_b/H_a$  in figure 3 d), we see how acceleration in the visible dimensions correlates with contraction of the extra dimensions. The Hubble parameter today in the model is somewhat low, which is related to the large value of  $\Omega_{m0}$ . At late times  $3H_{4D}^2 = \kappa^2 \rho_{m,in} a^{-3}$ , so we have  $\Omega_m = (H_a/H_{4D})^{-2} b^{-6}$ . In

order to get enough acceleration in the recent past, it seems that the extra dimensions must have recently collapsed, so  $b \approx 1$  today. The value  $\Omega_m = 0.3$ , for example, then requires  $H_a/H_{4D} = 1.8$ . The maximum value of  $H_a/H_{4D}$  in the best-fit model is only 1.3, and the value today is 1.2. Without the BBN constraint, the situation would be better, with higher values of  $H_a/H_{4D}$ .

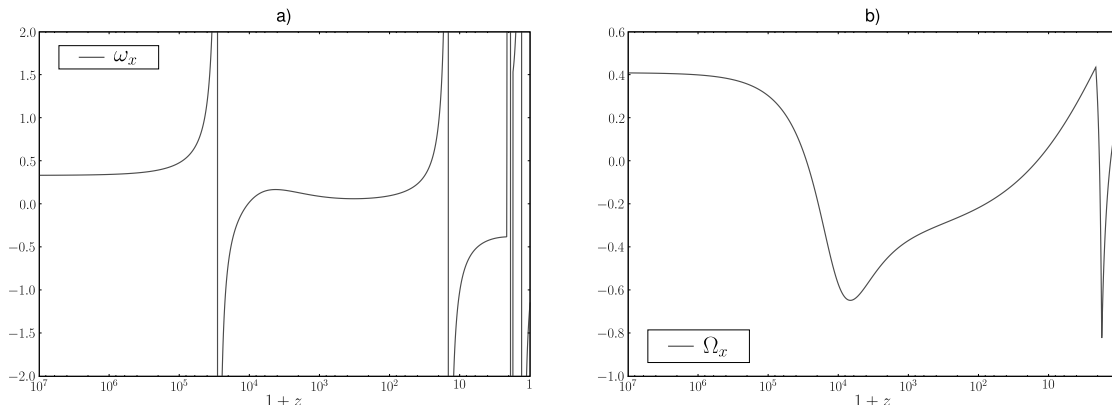
The quantity  $H_a/H_{4D}$  also gives the relation between the age of the universe and the present value of the Hubble parameter, since  $H_a/H_{4D} = 3H_a t/2$  at late times. A model-independent observational constraint on the age of the universe is given by the ages of globular clusters [40], which lead to the lower limit  $t_0 \geq 11.2$  Gyr at 95% C.L. and a best-fit age of  $t_0 = 13.4$  Gyr. The best model-independent measure of the current value of the Hubble parameter comes from the Hubble Key Project [41]. The result is sensitive to the treatment of Cepheids, and two different analyses yield  $H_{a0} = 0.73 \pm 0.06$  km/s/Mpc and  $H_{a0} = 0.62 \pm 0.05$  km/s/Mpc ( $1\sigma$  limits). Taking the best-fit value for  $t_0$  and the mean values for  $H_{a0}$  gives  $H_a/H_{4D} = 1.5$  and  $H_a/H_{4D} = 1.27$ , respectively. The value in the best-fit model is too low, but not drastically so, taking into account the uncertainties in  $t_0$  and  $H_{a0}$ .

**The effective equation of state.** One reason for the poor fit is the extra-dimensional modification of the relationship between the expansion rate and the distance in (2.9). Rapid oscillations of the Hubble parameter do not by themselves rule out the expansion history. If we take the expansion rate  $H_a$  for the string gas model and calculate the distance using the FRW relation (2.7), the  $\chi^2$  of the best fit without the BBN constraint improves by 4.2 points for the Union data and 30.7 points for the ESSENCE data, and the fits correspond to a probability of 29% and 20%, respectively.

The string gas cosmology context aside, this provides an interesting demonstration of how a model with an expansion history radically different from  $\Lambda$ CDM is consistent with the SNIa data. In figure 4 a), we plot the effective equation of state  $\omega_x$  of the best-fit model, defined by treating the string gas and the extra-dimensional geometrical contributions to the Friedmann equation as one effective component, so that (2.4) reads  $3H_a^2 = 8\pi G_N(\rho_{\gamma,in}a^{-4} + \rho_{m,in}a^{-3} + \rho_x)$ , with  $p_x$  defined correspondingly for (2.5), and  $w_x \equiv p_x/\rho_x$ . The variation in the effective equation of state  $w_x$  is extreme: in fact, the equation of state diverges, because  $\rho_x$  passes through zero (in the plot, we cut  $w_x$  off at  $\pm 2$ ). The equation of state is far from constant, and far from slowly varying, unlike assumed in most parametrisations. (For the importance of the assumed parametrisation of the equation of state for analysing the data, see [42].) Because  $w_x$  diverges, the evolution of the effective energy density is better displayed via the effective density parameter  $\Omega_x \equiv \kappa^2 \rho_x / (3H_a^2)$  shown in figure 4 b). The effective energy density is negative for a significant part of the evolution, as could be expected on the basis of the strong deceleration seen in figure 3 c).

## 4 Discussion

**Conclusions.** We have studied the late-time acceleration due to the oscillations of extra dimensions in string gas cosmology in the simple model discussed in [18]. We have fitted



**Figure 4.** a) Effective equation of state and b) effective density parameter, for the best-fit model to the Union data, with the BBN constraint.

the expansion history to the Union and ESSENCE sets of type Ia supernovae. The string gas model does not fit the SNIa data as well as the  $\Lambda$ CDM model. With the Union SNIa data, the difference in the goodness-of-fit is small, but the fit to the ESSENCE data is poor. Also, the best-fitting string gas models have a significant fraction of the energy density during BBN in the string gas, and taking into account the BBN constraint on new radiation degrees of freedom makes the fit worse. Further, we have considered a rather conservative BBN limit, allowing for neutrino chemical potential. Since the best-fit model is at the boundary of the region allowed by BBN, we would expect the fit to become worse as the BBN limit becomes more stringent. In the model, the matter density is also too high and the Hubble parameter today somewhat low, so taking further observational constraints into account would be likely to degrade the fit further. In any case, the model can still provide a stabilisation mechanism for the extra dimensions during the matter dominated era, for which it was originally introduced.

Leaving aside the physical origin of the oscillations and the constraint from BBN, the model demonstrates how an expansion history which is very different from the  $\Lambda$ CDM model, with strong oscillations of the Hubble parameter, can still provide a good fit to the supernova data. (In this context, it may be interesting that the Hubble parameter inferred from observations of the ages of passively evolving galaxies shows oscillations [32], though it is premature to draw strong conclusions from the data.) The fit only becomes poor when the change in the expansion rate-distance relationship due to the extra dimensions is taken into account. This in turn is a concrete example of a model where this FRW consistency condition, discussed in [24], is strongly violated.

**Improving the model.** As discussed in [18], the energy-momentum tensor for the string gas is expected to be more complex than (2.12)–(2.14). The energy density (2.12) corresponds to a gas of strings which all have the same momentum  $Ml_s/a$  in the visible dimensions, while a realistic gas would have a distribution of strings with different momenta. The evolution of terms with different values of  $M$  is qualitatively the same: they scale like radiation in the radiation-dominated era and start tracking the matter during the

matter-dominated era until the onset of oscillations. However, the different terms will lead to quantitatively slightly different oscillations, and as we have seen, the evolution is very sensitive to the parameters of the string gas. In order to explore this possibility, we would have to know the distribution of string momenta, which depends on how the string gas was created in the early universe and whether it has thermalised.

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